Characterizations and Recognition of Unit Circular-Arc Graphs

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ABSTRACT

A *circular-arc (CA)* graph G is the intersection graph of a family $\mathcal{A}$ of arcs of a circle C. In this case, the pair $(C, \mathcal{A})$ is said to be a *CA model for G*. One of the main subclasses of CA graphs are the *unit circular-arc graphs (UCA)*, which are formed by those graphs admitting a CA model whose arcs are all of the same size. Such a model is called a UCA model. In this talk we describe the characterizations and recognition algorithms for UCA known in the literature. In particular, we present a new characterization and recognition algorithm for the class. The motivation for this work has been a recent paper by Durán, Gravano, McConnell, Spinrad and Tucker in which they describe an algorithm of complexity $O(n^2)$ for recognizing whether a graph G with n vertices and m edges is a unit circular-arc (UCA) graph. Furthermore the following open questions were posed in this paper. (1) Is it possible to construct a UCA model for G in polynomial time ? (2) Is it possible to construct a UCA model, whose extremes of the arcs correspond to integers of polynomial size ? (3) If (2) is true, could such a model be constructed in polynomial time ? In the present talk, we describe a characterization of UCA graphs, based on network circulations. The characterization leads to a different recognition algorithm and to answering these questions, in the affirmative. We construct a UCA model whose extremes of the arcs correspond to integers of size $O(n)$. The proposed algorithms, for recognizing UCA graphs and constructing UCA models, have complexities $O(n+m)$. Finally, we also describe a linear time algorithm for finding feasible circulations in networks with non negative lower capacities and unbounded upper capacities. Such an algorithm is employed in the model construction for UCA graphs.