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ON SELF-CLIQUE GRAPHS WITH TRIANGULAR CLIQUES

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Abstract. A graph is an $\{r, s\}$-graph if the set of degrees of their vertices is $\{r, s\}$. A *clique* of a graph is a maximal complete subgraph. The *clique graph* $K(G)$ of a graph G is the intersection graph of all its cliques. A graph G is *self-clique* if G is isomorphic to $K(G)$. We show the existence of self-clique $\{5, 6\}$-graphs whose cliques are all triangles, thus solving a problem posed by Chia and Ong in [7].

1. Introduction.

Our graphs are simple and connected. A *clique* of a graph G is a maximal complete subgraph. The *clique graph* of G is the intersection graph $K(G)$ of the cliques of G, and G is *self-clique* if G and $K(G)$ are isomorphic. A graph is r-*regular* if every vertex has degree r; it is an $\{r, s\}$-*graph* if the set of degrees of its vertices is $\{r, s\}$. Clique graphs have been applied to study the Fixed Point Property for Posets [11] and to Loop Quantum Gravity [20–22]. The study of self-clique graphs began in [8] and has been pursued in [1–7, 12–15] among others. Related work may be found in [9, 10, 16–19, 23–25].

Chia and Ong [7] defined $\mathcal{G}(k)$, for $k \geq 2$, as the class of all self-clique graphs whose cliques all have k vertices. They classified the graphs in $\mathcal{G}(2)$ and started the study of $\mathcal{G}(3)$: They proved that any vertex of a graph in $\mathcal{G}(3)$ must have a degree in $\{2, 3, 4, 5, 6\}$. They proved that r-regular graphs in $\mathcal{G}(3)$ only exist when $r \in \{4, 5, 6\}$, classified the 4-regular graphs in $\mathcal{G}(3)$ and gave families of 5- and 6-regular graphs in $\mathcal{G}(3)$. Chia and Ong also showed that $\{r, s\}$-graphs in $\mathcal{G}(3)$ do not exist for $\{r, s\} = \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{4, 6\}$; gave a family of $\{4, 5\}$-graphs and an example of a $\{2, 6\}$-graph in $\mathcal{G}(3)$. But the existence of $\{5, 6\}$-graphs in $\mathcal{G}(3)$ remained as an open problem.

Here we solve the Problem (ii) in [7] by showing the existence of self-clique $\{5, 6\}$-graphs whose cliques are all triangles.

2. The examples

Theorem 2.1. *There exist self-clique* $\{5, 6\}$*-graphs whose cliques are all triangles.*

Proof. We provide adjacency lists for two examples in Tables 1 and 2. We also provide drawings of both examples in Figure 1. For clarity, some vertices have been drawn twice or even thrice: repeated occurrences of vertices are drawn with white interior. Cliques are also numbered in bold typeface in such a way that the required isomorphisms $\phi : G \to K(G)$ are just $\phi(x) = \boldsymbol{x}$. A direct inspection ends the proof. $\square$

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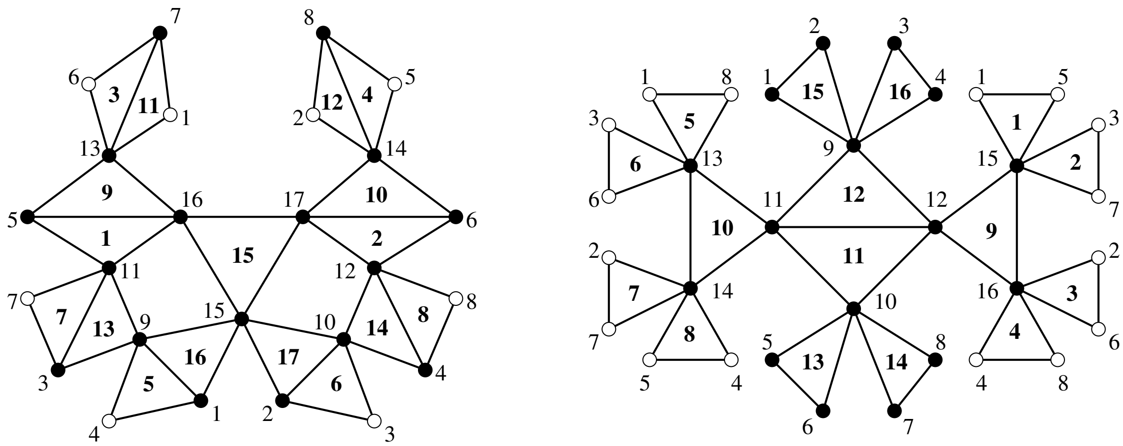

FIGURE 1. Crab and Nebula: Two self-clique $\{5, 6\}$-graphs whose cliques are all triangles. Vertices with the same labels in each drawing must be identified.

Vertex	Neighbors	Vertex	Neighbors	Vertex	Neighbors
1	4, 7, 9, 13, 15	7	1, 3, 6, 11, 13	13	1, 5, 6, 7, 16
2	3, 8, 10, 14, 15	8	2, 4, 5, 12, 14	14	2, 5, 6, 8, 17
3	2, 7, 9, 10, 11	9	1, 3, 4, 11, 15	15	1, 2, 9, 10, 16, 17
4	1, 8, 9, 10, 12	10	2, 3, 4, 12, 15	16	5, 11, 13, 15, 17
5	8, 11, 13, 14, 16	11	3, 5, 7, 9, 16	17	6, 12, 14, 15, 16
6	7, 12, 13, 14, 17	12	4, 6, 8, 10, 17		

TABLE 1. Adjacency list for Crab.

Vertex	Neighbors	Vertex	Neighbors	Vertex	Neighbors
1	2, 5, 8, 9, 13, 15	7	2, 3, 8, 10, 14, 15	13	1, 3, 6, 8, 11, 14
2	1, 6, 7, 9, 14, 16	8	1, 4, 7, 10, 13, 16	14	2, 4, 5, 7, 11, 13
3	4, 6, 7, 9, 13, 15	9	1, 2, 3, 4, 11, 12	15	1, 3, 5, 7, 12, 16
4	3, 5, 8, 9, 14, 16	10	5, 6, 7, 8, 11, 12	16	2, 4, 6, 8, 12, 15
5	1, 4, 6, 10, 14, 15	11	9, 10, 12, 13, 14		
6	2, 3, 5, 10, 13, 16	12	9, 10, 11, 15, 16		

TABLE 2. Adjacency list for Nebula.

References

[1] R. Balakrishnan and P. Paulraja. *Self-clique graphs and diameters of iterated clique graphs.* Utilitas Math. **29** (1986) 263–268.

[2] A. Bondy, G. Durán, M.C. Lin and J.L. Szwarcfiter. *A sufficient condition for self-clique graphs.* Electronic Notes on Discrete Mathematics **7** (2001) 19–23.

[3] A. Bondy, G. Durán, M.C. Lin and J.L. Szwarcfiter. *Self-clique graphs and matrix permutations.* J. Graph Theory **44** (2003) 178–192.

[4] F. Bonomo. *Self-clique Helly circular-arc graphs.* Discrete Math. **306** (2006) 595–597.

[5] G.L. Chia. *On self-clique graphs with given clique sizes.* Discrete Math. **212** (2000) 185–189.

[6] G.L. Chia and P.H. Ong. *On self-clique graphs with given clique sizes. II.* Discrete Math. **309** (2009) 1538–1547.

[7] G.L. Chia and P.H. Ong. *On self-clique graphs all of whose cliques have equal size.* Ars Combin. **105** (2012) 435–449.

[8] F. Escalante. *Über iterierte Clique-Graphen.* Abh. Math. Sem. Univ. Hamburg **39** (1973) 59–68.
[9] A.P. Figueroa and B. Llano. *An infinite family of self-diclique digraphs.* Appl. Math. Lett. **23** (2010) 630–632.
[10] M. Groshaus and L.P. Montero. *On the iterated biclique operator.* J. Graph Theory **73** (2013) 181–190.
[11] S. Hazan and V. Neumann-Lara. *Fixed points of posets and clique graphs.* Order **13** (1996) 219–225.
[12] F. Larrión, V. Neumann-Lara, M.A. Pizaña and T.D. Porter. *Self clique graphs with prescribed clique-sizes.* Congr. Numer. **157** (2002) 173–182.
[13] F. Larrión, V. Neumann-Lara, M.A. Pizaña and T.D. Porter. *Recognizing self-clique graphs.* Mat. Contemp. **25** (2003) 125–133.
[14] F. Larrión, V. Neumann-Lara, M.A. Pizaña and T.D. Porter. *A hierarchy of self-clique graphs.* Discrete Math. **282** (2004) 193–208.
[15] F. Larrión and M.A. Pizaña. *On hereditary clique-Helly self-clique graphs.* Discrete Appl. Math. **156** (2008) 1157–1167.
[16] M. Liverani, A. Morgana and C.P. de Mello. *The K-behaviour of p-trees.* Ars Combin. **83** (2007) 33–45.
[17] M. Matamala and J. Zamora. *A new family of expansive graphs.* Discrete Appl. Math. **156** (2008) 1125–1131.
[18] M. Milanič, A.S. Pedersen, D. Pellicer and G. Verret. *Graphs whose complement and square are isomorphic.* Discrete Math. **327** (2014) 62–75.
[19] E. Prisner. *Graph dynamics.* Longman, Harlow, 1995.
[20] M. Requardt. *(Quantum) spacetime as a statistical geometry of lumps in random networks.* Classical Quantum Gravity **17** (2000) 2029–2057.
[21] M. Requardt. Space-time as an order-parameter manifold in random networks and the emergence of physical points. In *Quantum theory and symmetries (Goslar, 1999)*, pages 555–561. World Sci. Publ., River Edge, NJ, 2000.
[22] M. Requardt. *A geometric renormalization group in discrete quantum space-time.* J. Math. Phys. **44** (2003) 5588–5615.
[23] J.L. Szwarcfiter. A survey on clique graphs. In B.A. Reed and C. Linhares-Sales, editors, *Recent advances in algorithms and combinatorics*, volume 11 of *CMS Books Math./Ouvrages Math. SMC*, pages 109–136. Springer, New York, 2003.
[24] B. Zelinka. *Periodicity of graph operators.* Discrete Math. **235** (2001) 349–351. Combinatorics (Prague, 1998).
[25] B. Zelinka. *On a problem of E. Prisner concerning the biclique operator.* Math. Bohem. **127** (2002) 371–373.

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